

## Tunneling into a nonequilibrium Luttinger liquid with impurity

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We evaluate tunneling rates into/from a voltage ( $U$ ) biased quantum wire containing a weak backscattering defect. Interacting electrons in such a wire form a true *nonequilibrium* state of the Luttinger liquid (LL). This state is created due to inelastic electron backscattering leading to the emission of nonequilibrium plasmons with typical frequency  $\hbar\omega \leq U$ . The tunneling rates are split in two edges. The tunneling exponent at the Fermi edge is *positive* and equals that of the equilibrium LL, while the exponent at the side edge  $E_F - U$  is *negative* if Coulomb interaction is not too strong.

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By virtue of advances in modern nanotechnology electron tunneling spectroscopy became a powerful technique that enables to reveal electron correlations on a mesoscale. Suppression or enhancement of the tunneling conductance at low bias is a signature of electron interaction in the system and is commonly called a “zero-bias anomaly” (ZBA).<sup>1,2</sup> Measurements of ZBA in disordered metals,<sup>3</sup> in high-mobility two-dimensional electron gases,<sup>4</sup> in the edges of quantum Hall systems,<sup>5</sup> and recent measurements of magnetotunneling in arrays of quantum wires<sup>6</sup> are milestones of this field.

Of particular interest on this way is the study of electron tunneling into quantum nanowires.<sup>6–8</sup> Central to much of the fascinating physics of these one-dimensional (1D) electron systems is that Coulomb interaction has a dramatic effect leading to the emergence of the Luttinger liquid (LL).<sup>9</sup> This strongly correlated state of matter is commonly described in terms of bosonic elementary excitations. Measurements of tunneling in nanowires confirm predictions based on the LL model. It has been understood as well, how the presence of impurities and system boundaries affects ZBA in the equilibrium LL.<sup>10–12</sup>

Notably, the behavior of a strongly correlated quantum system can change drastically when it is driven out of equilibrium. Remarkable examples include the Kondo phenomena<sup>13</sup> and the Fermi-edge singularity problem.<sup>14</sup> Recent experiments initiate the study of the nonequilibrium tunneling spectroscopy of carbon nanotubes<sup>15</sup> and quantum Hall edges.<sup>16</sup>

In this Rapid Communication we consider the tunneling into a voltage biased one-channel ballistic wire, containing a weak backscattering defect (Fig. 1). Previous studies of this model focused on the nonlinear conductance and shot noise.<sup>11,17</sup> However, the tunnel spectroscopy of this problem, which requires the analysis of the single-particle Green’s function, has never been addressed.<sup>18</sup> We show that interacting electrons in such a wire form a generic nonequilibrium LL state, characterized by *non-Gaussian* plasmon correlations, and develop a real-time instanton approach to evaluate the tunneling rates. Inelastic electron backscattering at the defect induces the emission of *real* nonequilibrium plasmons with typical frequencies  $\hbar\omega \leq U$ .<sup>19,20</sup> In the nondissipative LL they transfer the shot noise of backscattered current with the *Poissonian* statistics to the distant point of tunneling  $\bar{x}$

(Fig. 1), thereby considerably influencing the ZBA. Let us emphasize an important difference between the present setup and that of Ref. 21. While in the model of Ref. 21 the nonequilibrium state is “injected” into the LL, here it is created by a scatterer located *inside* the LL.

Our results can be summarized as follows. We consider a spinless LL with  $e$ - $e$  interaction described by the conventional parameter  $K$ . The tunneling rates into/from the right electron states ( $R$  states) in the LL are split in two edges,  $\Gamma_R^\pm(\epsilon) = \Gamma_0^\pm(\epsilon) + \Delta\Gamma^\pm(\epsilon)$ , as shown in Fig. 2. The first term here accounts for the equilibrium contribution to ZBA around the Fermi energy,  $\Gamma_0^\pm(\epsilon) \propto \theta(\pm\epsilon)|\epsilon/E_F|^{2\gamma}$ , with exponent  $\gamma = (1-K)^2/4K$ . The singular part of the nonequilibrium correction at  $\epsilon = -U$  behaves as

$$\Delta\Gamma_R^-(\epsilon) \propto -r_U^2 |\epsilon + U|^{2(\gamma-\delta)} \sin \begin{cases} 2\pi\delta, & \epsilon > -U \\ 2\pi\gamma, & \epsilon < -U \end{cases} \quad (1)$$

in case of tunneling from the  $R$  state into the tip and

$$\Delta\Gamma_R^+(\epsilon) \propto r_U^2 \theta(\epsilon + U) |\epsilon + U|^{2(\gamma-\delta)} \quad (2)$$

in case of tunneling from the tip into the  $R$  state. Here  $\delta = (1-K)/2$  and  $r_U^2 = r_0^2 (E_F/U)^{4\delta} / \Gamma(2K) \ll 1$  is the renormalized reflection coefficient.<sup>22</sup> The tunneling rates into or from the left electron branch remain almost equal to the equilibrium ones,  $\Gamma_L^\pm(\epsilon) \approx \Gamma_0^\pm(\epsilon + U)$ . The above power-law singularities are smeared on a scale of the nonequilibrium dephasing rate

$$\tau_\phi^{-1} = (2/\pi) U r_U^2 \sin^2 \pi\delta. \quad (3)$$

It is worth stressing the oscillatory dependence of  $1/\tau_\phi$  on the interaction parameter  $K$ . It differs from that obtained in the model of Ref. 21, which reflects a different type of the nonequilibrium LL state.

Result (2) corresponds to inelastic tunneling with absorption of real plasmons. An electron tunneling into the LL with the energy  $\epsilon < 0$  can accommodate itself above the Fermi energy of right moving states by picking up the energy  $\hbar\omega > |\epsilon|$  from the nonequilibrium plasmon bath. Since the energy of out-of-equilibrium plasmons is limited by the applied voltage, one has a threshold:  $\epsilon > -U$ . The correction [Eq. (1)] describes the inverse processes—inelastic tunneling from the

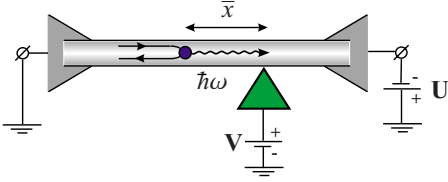


FIG. 1. (Color online) Tunneling experiment with a voltage biased quantum wire ( $\bar{x} > 0$ ). Right moving electrons have a larger chemical potential relative to left moving electrons,  $\mu_R - \mu_L = U > 0$ .

LL accompanied by the stimulated emission of nonequilibrium plasmons with typical energy  $\hbar\omega \approx U$ . In the absence of interaction the splitting of tunneling rates  $\Gamma_R^\pm(\epsilon)$  can be understood as the result of the double-step distribution of  $R$  states due to the scattering off the impurity.<sup>23</sup>

We model the wire as the spinless LL (Ref. 9) with linear dispersion and short-range forward  $e-e$  interaction with the amplitude  $V_0$ , which fixes  $K = (1 + V_0/\pi v_F)^{-1/2}$ . The electron field is decoupled into right- and left-moving fields  $\psi_\pm$ . A weak impurity at  $x=0$  is characterized by a bare reflection amplitude  $r_0 \ll 1$ .

Our theoretical analysis is based on the functional bosonization (FB).<sup>12</sup> We consider an electron motion in a Hubbard-Stratonovich field  $\varphi$  which mediates  $e-e$  interaction. The special feature of 1D geometry is that by a local gauge transformation,  $\psi_\eta \rightarrow e^{i\vartheta_\eta} \psi_\eta$ , the coupling between  $\psi_\pm$  and  $\varphi$  can be removed everywhere except the points of scattering provided that  $\partial_\eta \vartheta_\eta = -\varphi$ , where  $\partial_\eta = \partial_t + \eta v_F \partial_x$ . The phases  $\vartheta_\pm$  define charge and current responses to the potential  $\varphi$  in each chiral branch,  $\rho_\eta = \partial_x \vartheta_\eta / 2\pi$  and  $j_\eta = -\partial_t \vartheta_\eta / 2\pi$ . Following this gauge transformation one constructs the bosonized Keldysh action  $\mathcal{S} = \mathcal{S}_b + \mathcal{S}_{\text{imp}}$  in terms of the variables  $\vartheta$  and  $\phi$  (Ref. 19): (see also a related work<sup>24</sup> on multichannel wires)

$$\begin{aligned} \mathcal{S}_b = & \int_{\mathcal{C}} dt \int dx \sum_{\eta=\pm} v \left[ \frac{1}{2} \dot{\vartheta}_\eta^2 + \frac{\eta v_F}{2} (\partial_x \vartheta_\eta) \dot{\vartheta}_\eta + \varphi \dot{\vartheta}_\eta \right] \\ & + \frac{1}{2} \varphi \left( V_0^{-1} + \frac{1}{\pi v_F} \right) \varphi, \\ \mathcal{S}_{\text{imp}} = & -\frac{i}{4} |r_0|^2 \text{Tr} [\hat{g}_+ e^{i\hat{\Phi}} \hat{g}_- e^{-i\hat{\Phi}}]_{x=0}. \end{aligned} \quad (4)$$

Here  $\nu = (2\pi v_F)^{-1}$  is the 1D density of states,  $\Phi = \vartheta_- - \vartheta_+$ , and  $\hat{g}_\pm$  are “quasiclassical” Green’s function in the right or left leads. They are fixed by boundary conditions  $\hat{g}_\eta = (1 - 2f_\eta)(\hat{\tau}_3 + i\hat{\tau}_2) - \hat{\tau}_1$ ,  $f_\pm(t_1 - t_2)$  being the electron distribution functions. In the limit of zero-temperature one has  $f_\pm(t) = ie^{-i\mu_\pm t} / 2\pi(t + i0)$ , where the chemical potentials satisfy  $\mu_+ - \mu_- = U$ .<sup>25</sup> The diagonal matrix  $\hat{\Phi} = \text{diag}(\Phi^-, \Phi^+)$  and the Pauli matrices  $\hat{\tau}$  act in the Keldysh space, the upper indices  $\pm$  referring to the two branches of the Keldysh contour  $\mathcal{C}$ . The trace (Tr) operation is performed in the Keldysh  $\times$  time space.

We start by considering Gaussian charge and current fluctuations described by the action  $\mathcal{S}_b$ . Within the FB electron

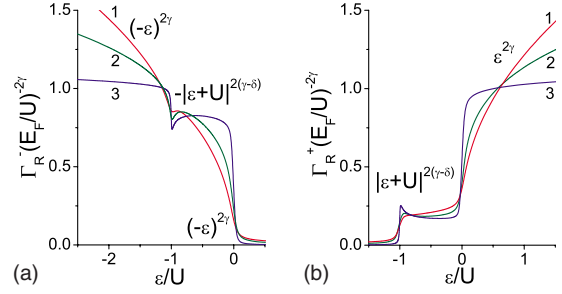


FIG. 2. (Color online) Energy dependence of tunneling rates from the nonequilibrium LL (left pane) and into the LL (right pane), shown for  $r_0^2 = 0.2$  and different strengths of repulsive  $e-e$  interaction: (1)  $K=0.4$ ; (2)  $K=0.5$ ; and (3)  $K=0.75$ .

phases  $\vartheta_\pm$  have no free dynamics—in contrast to the conventional bosonization—but rather respond to the internal electric field. This response is found by optimizing  $\mathcal{S}_b$  for a given  $\varphi$ , which gives the gauge relation  $\partial_\eta \vartheta_\eta = -\varphi$ . One has to solve it by taking the proper structure of the Keldysh theory into account,  $\hat{\vartheta}_\eta[\varphi] = -\hat{D}_{0\eta}^{-1} \hat{\tau}_3 \hat{\varphi}$ . We have introduced doublets, e.g.,  $\hat{\varphi} = (\varphi^-, \varphi^+)^T$ , and the bare particle-hole propagator

$$D_{0\eta}^\pm(t, x) = v_F^{-1} n_B^\pm(t - \eta x/v_F), \quad (5)$$

where  $n_B^\pm(t) = -i/2\pi(t \mp ia)$  and  $a \sim E_F^{-1}$  is a short time cut-off. Then the quadratic action  $\mathcal{S}_b$ , expressed solely in terms of  $\varphi$ , assumes the random-phase approximation (RPA) form,  $\mathcal{S}_b = \frac{1}{2} \hat{\varphi}^T \hat{V}^{-1} \hat{\varphi}$ , with a nonlocal effective interaction  $\hat{V} = V_0 - (V_0^2/\pi) K^2 \partial_x \hat{D}$ . Here  $\hat{D} = \frac{1}{2} \sum_{\mu\lambda} \mu \hat{D}_{\mu\lambda}$  and  $\hat{D}_{\mu\lambda}$  is the propagator of the plasmon modes moving with velocity  $u = v_F/K$ ,

$$D_{\mu\lambda}^\pm(t, x) = \frac{1}{v_F} \left[ c_{\mu\lambda}^+ n_B^\pm \left( t - \frac{\mu x}{u} \right) + c_{\mu\lambda}^- n_B^\pm \left( t + \frac{\mu x}{u} \right) \right], \quad (6)$$

where  $c_{\mu\mu}^+ = 1 + \gamma$ ,  $c_{\mu\mu}^- = \gamma$ , and  $c_{\mu, -\mu}^\pm = (1 - K^2)/(4K)$ .

To find the tunneling rates we represent the electron Green’s function at the point of tunneling  $\bar{x} > 0$  as a path integral over the field  $\varphi$ ,

$$G_\eta^\pm(\bar{x}, \tau) = \int \mathcal{D}\varphi e^{i\mathcal{S}_J^\pm[\varphi] + i\mathcal{S}[\varphi]} G_\eta^\pm(\bar{x}, \tau, 0; [\varphi]). \quad (7)$$

Here  $\hat{G}_\eta^\pm(\bar{x}, t, t'; [\varphi])$  denotes the Green’s function for a given configuration of  $\varphi$ . It satisfies the Dyson equation with the spatially local self-energy

$$\hat{\Sigma}_\eta[\varphi] = -i\delta(x)(|r_0|^2 v_F/2) e^{i\eta\hat{\Phi}} \hat{g}_- e^{-i\eta\hat{\Phi}}$$

due to impurity scattering, the phase  $\hat{\Phi}(t, [\varphi])$  being the linear functional of  $\varphi$  introduced above. The action  $\mathcal{S}_J^\pm$  describes the creation of a hole at time  $t=0$  and an electron at the instant  $t=\tau$ , while  $\mathcal{S}_J^\pm$  corresponds to the inverse process,

$$\mathcal{S}_J^\pm(\tau, \bar{x}, [\varphi]) = \vartheta_\eta^\pm(\bar{x}, \tau) - \vartheta_\eta^\pm(\bar{x}, 0) = - \int dt dx \hat{\vartheta}_\eta^T \vec{J}_\pm. \quad (8)$$

The source here, e.g.,  $\vec{J}_> = \delta(x - \bar{x})(\delta(t), -\delta(t - \tau))^T$ , acts on both branches of the Keldysh contour.

To find the Green’s function we proceed with a semiclas-

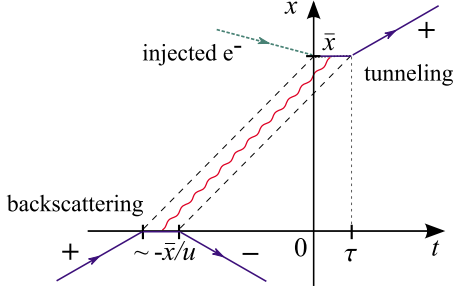


FIG. 3. (Color online) *Real plasmons* (wavy line) created in the course of inelastic electron backscattering at  $t \sim -\bar{x}/u$  are absorbed by the injected electron at  $t \sim 0$ ; typical duration of a scattering/tunneling event is  $\Delta t \sim \tau \ll \bar{x}/u$ .

sical approximation.<sup>26</sup> One looks for a saddle-point trajectory  $\varphi^*$  which optimizes the total action  $\mathcal{S}_{\text{tot}} = \mathcal{S} + \mathcal{S}_I$  and further estimates the tunneling rate by evaluating  $\mathcal{S}_{\text{tot}}[\varphi^*]$ . For  $r_U^2 \ll 1$  we can find such a trajectory approximately, minimizing only the quadratic part of the action,  $S_0 = S_b + S_j$ . This results in  $\tilde{\vartheta}_\mu[\varphi^*] = \hat{D}_{\mu\eta} \vec{J}$ . Here the phase-phase correlation function satisfies the relations  $\nu \partial_t \hat{D}_{\mu\lambda} = \hat{D}_{0\mu} \delta_{\mu\lambda} - \hat{D}_{\mu\lambda}$  and  $\hat{D}_{\mu\lambda}(0, 0) = 0$ , that enables easy evaluation of  $\tilde{\vartheta}^*$  using the Eqs. (5) and (6). For instance, in case of tunneling from the tip into the right branch ( $\eta = +$ ) the relative phases  $\tilde{\Phi}_* = \tilde{\vartheta}_-^* - \tilde{\vartheta}_+^*$  explicitly read

$$i\Phi_*^\pm(t) = \ln \left[ \left( \frac{t + \bar{x}/u \mp ia}{t - \tau + \bar{x}/u \mp ia} \right)^{1-\delta} \times \left( \frac{t - \tau - \bar{x}/u + ia}{t - \bar{x}/u - ia} \right)^\delta \left( \frac{t - \tau + \bar{x}/v_F \mp ia}{t + \bar{x}/v_F \mp ia} \right) \right]. \quad (9)$$

Substituting  $\varphi^* = -\partial_\eta \tilde{\vartheta}_\eta^*$  into the RPA action we obtain  $i\mathcal{S}_0^\pm(\tau) = -2\gamma \ln(\pm i\tau/a)$ . The impurity action evaluated on the instanton (9) consists of four terms,  $\mathcal{S}_{\text{imp}}^*(\tau) = \sum_{\alpha\beta} \mathcal{S}_{\text{imp}}^{\alpha\beta}(\tau)$ , the indices  $\alpha, \beta = \pm$  arising from the Keldysh structure of  $\tilde{\Phi}_*$ . At  $U > 0$  the main contribution is given by

$$\mathcal{S}_{\text{imp}}^+ = \frac{i|r_0|^2}{4\pi^2} \int \int d^2t \frac{e^{-i\Phi_*^-(t_1) + i\Phi_*^+(t_2) - iU(t_1 - t_2)}}{(t_1 - t_2 + ia)^{2(1-2\delta)}}. \quad (10)$$

Here we have modified the  $1/t^2$  behavior of the bare equal point polarization operator by taking into account the time-dependent LL renormalization of the reflection amplitude  $r(t) \sim r_0(t/a)^{2\delta}$ , which results from quantum fluctuations around the saddle-point trajectory  $\varphi^*$ .

We further concentrate on the limit of long  $\bar{x}$  so that  $\tau_* \ll \bar{x}/v_F - \bar{x}/u$ , where  $\tau_* = \max\{\tau_\phi, 1/\Delta\epsilon\}$  is the typical accommodation time and  $\Delta\epsilon = \min\{|\epsilon|, |U + \epsilon|\}$  is the energy relative to the nearest edge. Under this condition the Fabry-Perot oscillations in  $\Gamma_\epsilon^\pm$  (Refs. 27 and 28) are suppressed. Then instanton (9) consists of well separated plasmon and particle-hole kinks, moving with velocities  $u$  and  $v_F$ , and gives two independent contributions,  $\mathcal{S}_p^*$  and  $\mathcal{S}_{\text{ch}}^*$ , to the impurity action. Their long-time behavior at  $\tau \gg 1/U$  is defined by singularities of integrand (10):

$$i\mathcal{S}_p^*(\tau) \approx r_U^2 \mathcal{C}(iU\tau)^{2\delta} e^{iU\tau} - |\tau|/2\tau_\phi,$$

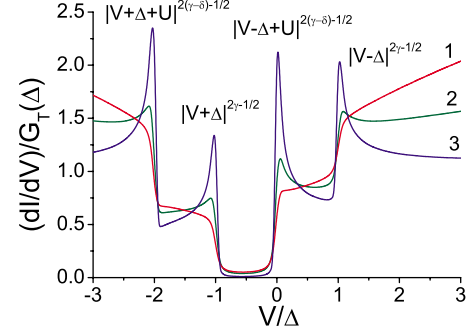


FIG. 4. (Color online) Differential tunnel conductance between the BCS tip and the LL. The parameters are:  $U = \Delta$ ,  $r_U^2 = 0.2$ , (1)  $K = 0.4$ ; (2)  $K = 0.5$ ; and (3)  $K = 0.75$ .

$$i\mathcal{S}_{\text{ch}}^*(\tau) \approx r_U^2 (-iU\tau)^{4\delta} e^{-iU\tau} \Gamma(2K), \quad (11)$$

where the rate  $\tau_\phi^{-1}$  is given by Eq. (3) and the numerical factor  $C = \Gamma(2K)/\Gamma^2(1 + \delta)$ . This asymptotics is identical in both cases of tunneling into and from the LL. The linear growth of  $\mathcal{S}_p^*(\tau)$  stems from the time domain  $|t_{1,2}| \leq \tau$ , while the oscillations  $\propto \tau^{2\delta} e^{iU\tau}$  are governed by the far distant times  $|t_{1,2} + \bar{x}/u| \leq \tau$ . The action  $\mathcal{S}_p^*$  reveals the Poissonian statistics of the shot noise of backscattered current carried by plasmons (Fig. 3); analogously,  $\mathcal{S}_{\text{ch}}^*$  describes the shot noise due to excited electron-hole pairs at the distant times  $|t_{1,2} + \bar{x}/v_F| \leq \tau$ .

Within the saddle-point approximation we obtain from representation (7)

$$G_+^{\pm}(\bar{x}, \tau) \approx -i\pi\nu g_+^{\pm}(\tau) e^{i\mathcal{S}_0^\pm(\tau) + i\mathcal{S}_p^*(\tau) + i\mathcal{S}_{\text{ph}}^*(\tau)}. \quad (12)$$

Tunneling rates are related to  $G_+^{\pm}(\bar{x}, \tau)$  by the Fourier transformation. Since  $r_U^2 \ll 1$ , one can expand the oscillatory part of the action in Eq. (12), retaining only the first term. Then the particle-hole contribution  $\mathcal{S}_{\text{ch}}^*$  exactly cancels the first-order impurity correction to the Green's function,  $\Delta \hat{G}_+ = \langle \hat{G}_{0,+} \hat{\Sigma}_+(\varphi) \hat{G}_{0,+} e^{i\mathcal{S}} \rangle_\varphi$ , where the average is performed with the RPA action  $\mathcal{S}_b$ . Keeping the plasmon contribution to the tunneling action, we finally find

$$\Gamma_R^\pm(\epsilon) = \pm \frac{1}{\pi} (U/E_F)^{2\gamma} \Gamma(-2\gamma) \text{Im}\{(\mp z)^{2\gamma} + r_U^2 \mathcal{C}(\pm 1)^{2\gamma} \times \Psi(-2\gamma, 1 + 2\delta - 2\gamma, -1 - z)\}, \quad (13)$$

where  $z = (\epsilon + i/2\tau_\phi)/U$  is the complex energy and  $\Psi$  is the confluent hypergeometric function.<sup>29</sup> The latter is singular at  $z \rightarrow -1$ , yielding the power laws stated in Eqs. (1) and (2). We plot the rates [Eq. (13)] in Fig. 2 versus energy  $\epsilon$  for different strengths of  $e$ - $e$  interaction, indicating the edge exponents. Remarkably, in the vicinity of the edge  $\epsilon = -U$  the in-rate  $\Gamma_R^+$  is enhanced, while the out-rate  $\Gamma_R^-$  is suppressed, provided the nonequilibrium exponent  $\lambda = 2(\gamma - \delta)$  is *negative*, which is the case of not too strong interaction realized at  $K > \frac{1}{3}$ .

We illustrate our theory by considering tunneling in the LL from a superconducting tip with the singular BCS density of states  $\nu_t(\epsilon) \propto |\epsilon|(\epsilon^2 - \Delta^2)^{-1/2}$ . Current measurements using this setup enable to reveal the nonequilibrium structure in the

tunneling rates.<sup>15</sup> For the tunneling current we have

$$I = G_T \int d\varepsilon [f_{\varepsilon-V} \Gamma_{\varepsilon}^+ - (1 - f_{\varepsilon-V}) \Gamma_{\varepsilon}^-] \nu_1(\varepsilon - V), \quad (14)$$

where  $\Gamma_{\varepsilon}^{\pm} = \Gamma_R^{\pm}(\varepsilon) + \Gamma_L^{\pm}(\varepsilon)$  and  $G_T$  is the bare tunnel conductance. In Fig. 4 we show the differential conductance  $dI/dV$  in units of the normal-state conductance  $G_T(\Delta) \sim G_T(\Delta/E_F)^{2\gamma}$  at the scale  $\Delta$ . Due to the double-edge structure of the tunneling rates [Eq. (13)] the peaks of the BCS density of states are split by the bias voltage  $U$  and show power-law behavior with exponents  $\lambda_1 = 2\gamma - \frac{1}{2}$  and  $\lambda_2 = 2(\gamma - \delta) - \frac{1}{2}$ . Singularities of  $\nu_1(\varepsilon)$  visibly enhance the nonequilibrium structures in the rates  $\Gamma_{\varepsilon}^{\pm}$ , making the conductance profile strongly asymmetric, even in the limit of small  $r_U^2$ .

To summarize we have developed a real-time instanton approach to the problem of tunneling into the nonequilibrium

state of the interacting quantum wire containing a weak backscattering defect. Tunneling rates are split in two edges, the power-law exponent  $\lambda$  at the nonequilibrium edge  $\varepsilon = -U$  being *negative*, provided the repulsive  $e$ - $e$  interaction is not too strong ( $K > \frac{1}{3}$ ). This nonequilibrium effect is associated with inelastic electron tunneling accompanied by absorption or emission of real plasmons with a typical frequency  $\hbar\omega \sim U$ . The approach developed in this work will be useful for analysis of tunneling and interference in a broad class of nonequilibrium LL structures with impurities and/or tunneling couplings.

*Note added in proof.* Recently, a preprint appeared<sup>30</sup> on spectroscopy of LL driven out of equilibrium by local injection of hot electrons.

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